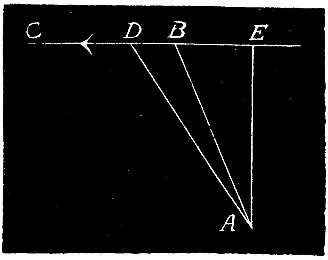

$AE'=AB\cos x$, or $b=8a\cos x$, $a=\dfrac{b}{8\cos x}$. $CD=10a$.

$DE=b\tan\theta$, $BE=b\tan x$. $y=\dfrac{10b}{8\cos x}+b\tan\theta-b\tan x$. $\dfrac{dy}{dx}=\dfrac{5b\sin x}{4\cos^2 x}-b\sec^2 x$.

Let $\dfrac{dy}{dx}=0$, whence $\sin\theta=\dfrac{4}{5}$, $\theta=53^\circ\,8'$.

The equation shows that b and θ do not affect the result.

Also solved by *Professors PHILBRICK, WHITAKER, ZERR*, and the *PROPOSER*.

14. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Right triangles are inscribed in a circle whose center $=(a, b)$, and radius$=c$. If one of the legs passes through a fixed point, prove that $c^2(x^2+y^2)=(a^2+b^2-c^2-ax-by)^2$ is the curve to which the other leg is always tangent; the fixed point being the origin of the co-ordinates.

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let $lx+my=1\ldots.(1)$ be the chord of the circle whose envelope is to be found. Then $y=\dfrac{m}{l}x\ldots.(2)$ is the side of the triangle passing through the fixed point, and $(x-a)^2+(y-b)^2=c^2\ldots.(3)$ is the equation to the circle.

(1) and (2) intersect in $x'=\dfrac{l}{l^2+m^2}$, $y'=\dfrac{m}{l^2+m^2}$.

This point being on (3), we must have on substituting x' and y' in (3), after reducing, $(a^2+b^2-c^2)(l^2+m^2)-2al-2bm+1=0\ldots.(4)$. Making (4) homogeneous in l and m by means of (1) and arranging,

$$[x^2-2ax+(a^2+b^2-c^2)]\frac{l^2}{m^2}+2(xy-bx-ay)\frac{l}{m}+[y^2-2by+(a^2+b^2-c^2)]=0\ldots.(5),$$

a quadratic in the parameter $\dfrac{l}{m}$, giving the envelope

$$c^2(x^2+y^2)=[(a^2+b^2-c^2)-by-ax]^2\ldots.(6).$$

Also solved by *P. S. BERG, G. B. M. ZERR* and the *PROPOSER*.

15. Proposed by CHARLES E. MYERS, Canton, Ohio.

From a given quantity of material a cylindrical cup with circular bottom and open top is to be made, the cup to contain the greatest amount. What must be its dimensions?

Solution by F. P. MATZ, M. S., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Represent the radius of the base by x, and the altitude by y; then, obviously, $\pi x^2+2\pi xy=s$. $\therefore y=(s-\pi x^2)\div 2\pi x$.

Also, $V=\pi x^2 y=\pi x^2\left(\dfrac{s-\pi x^2}{2\pi x}\right)$, $=$a maximum.